Online-Appendix to “Quantity restrictions on advertising, Commercial media bias, and Welfare”

Anna Kerkhof*          Johannes Münster+ 

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Abstract

This Online-Appendix to our paper “Quantity restrictions on advertising, Commercial media bias, and Welfare” contains: 1) additional background material, 2) a more detailed discussion of the microfoundations, 3) extensions of our model to deceptive advertising and sector specific regulation, 4) robustness checks concerning advertising demand and the model of television viewing, and 5) the omitted steps from the proof of Proposition 1.

1 Additional background material

*Media reports impact firm profits. We start by pointing to evidence that media reports impact firm profitability. For example, when the New York Times reported about a potential breakthrough in cancer research in its Sunday edition, it induced a permanent rise in share prices of EntreMed, a biotechnology company with licensing rights to the breakthrough - even though the information had already been published in Nature and various newspapers several months earlier (Huberman and Regev 2001). Similarly, Engelberg and Parsons (2011) show that media reporting has a causal effect on investor behavior, and Liu et al. (2014) show that media coverage of IPOs has long-run effects on the stock’s value. Media reporting may also bring public scrutiny to sensitive issues and lead to regulatory threats to firms. For example, Erfele and McMillan (1990) show that during the 1979 oil crisis, television reports on the oil crisis influenced home heating oil price ratios, but not residual fuel oil price ratios, and argue that the different reactions are explained by the threat of government intervention.

Moreover, several studies have shown that critical media reports impact consumer behavior. Niederdeppe and Frosch (2009) provide evidence that news coverage on trans fat reduced sales of trans-fat-products. Schlenker and Villas-Boas (2009) find a significant decrease in beef sales following reports on mad cow disease. Wakefield et al. (2003) show that antismoking messages can lower youth smoking rates. Laugesen and Meads (1991) find that doubling the coverage of smoking issues in newspapers lowers cigarette consumption as much as a 10% price increase.

Advertisers influence editors. Several recent papers have shown econometrically that advertisers systematically influence media content. The papers listed in Table 1 compare advertiser

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*University of Cologne, Albertus-Magnus Platz, D-50923 Cologne, Germany, e-mail: anna.kerkhof@wiso.uni-koeln.de.
+University of Cologne, Albertus-Magnus Platz, D-50923 Cologne, Germany, phone: +49 221 4704411, e-mail: johannes.muenster@uni-koeln.de.
spending with media coverage or slant. An empirical challenge is to identify whether there is a causal effect of advertising on media content. The recent literature has used state-of-the-art instrumental variable techniques and natural experiments to overcome this challenge (e.g. Gurun and Butler 2012).

Interviews and surveys of key players in the market also confirm that advertisers influence media content. An early survey by Soley and Craig (1992) on newspaper editors found that almost 9 out of 10 of their correspondents claim that advertisers have attempted to take influence on editorial decisions. In a survey of journalists by the Pew Research center (2000), a third of the journalists stated an “intrusion of commercial interests” into editorial decisions (p. 3). Similarly, An and Bergen’s survey (2007) on 219 advertising directors at US daily newspapers reports frequent conflicts between the journalism side and the business side of newspapers. Further evidence of advertisers’ influence comes from a content analysis of ostensibly noncommercial newscasts (Upshaw et al. 2007): 90% of the stations studied contained at least one instance per newscast of commercial messages outside regular commercial blocks.

<table>
<thead>
<tr>
<th>Study</th>
<th>Media</th>
<th>Advertiser</th>
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<tr>
<td>Warner et al. 1992</td>
<td>magazines</td>
<td>tobacco</td>
<td>US</td>
<td>“strong statistical evidence that cigarette advertising (...) diminished the coverage of the hazards of smoking” (p. 305)</td>
</tr>
<tr>
<td>Reuter &amp; Zitzewitz 2006</td>
<td>personal finance publications</td>
<td>mutual funds</td>
<td>US</td>
<td>“mutual fund recommendations are correlated with past advertising” (p. 197)</td>
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<td>*</td>
<td>newspapers</td>
<td></td>
<td>US</td>
<td>“but not in two national newspapers” (p. 197)</td>
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<tr>
<td>Reuter 2009</td>
<td>wine publications</td>
<td>wineries</td>
<td>US</td>
<td>“Wine Spectator appears largely to insulate reviewers from the influence of advertisers” (p. 125)</td>
</tr>
<tr>
<td>Rinallo &amp; Basuroy 2009</td>
<td>newspapers, magazines</td>
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<td>“there is evidence of a strong positive influence of advertising on coverage (...) exist in both Europe and the US” (p. 33)</td>
</tr>
<tr>
<td>Gambaro &amp; Puglisi 2015</td>
<td>newspapers</td>
<td>all</td>
<td>I</td>
<td>“coverage of a given company is positively related with the amount of ads purchased on that newspaper by that company” (p. 1)</td>
</tr>
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<td>DiTella &amp; Francescelli 2011</td>
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<td>“One standard deviation increase in monthly government advertising is associated with a reduction in the coverage of the government’s corruption scandals of (...) 18% of a standard deviation in coverage.” (p. 119)</td>
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<td>DeSmet &amp; Vanormelingen 2012</td>
<td>newspapers</td>
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<td>B</td>
<td>“advertisers in Belgian Dutch-language newspapers receive a significantly higher coverage” (p. 1)</td>
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<td>Gurun &amp; Butler 2012</td>
<td>local newspapers</td>
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<td>“positive slant about local companies is strongly positively related to the local advertising budget of these companies” (p. 563)</td>
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<tr>
<td>Dewenter &amp; Heimeshoff 2014</td>
<td>car magazines</td>
<td>car producers</td>
<td>G</td>
<td>“evidence for media bias in test scores” (p. 17)</td>
</tr>
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Table 1: Evidence on advertisers’ influence on media content
Conflict of interest between viewers and advertisers. At the center of our model is a conflict of interest between viewers and advertisers over media content. Here we provide additional empirical evidence that motivates this assumption, and in particular deepen the issue that viewers but not advertisers may favor accurate reporting of defects, risks, or negative externalities of products.

As we mention in the paper, two important cases of commercial media bias concern the health risks of tobacco and anthropogenic climate change. Reporting about the health risks of smoking is an important and well-documented case of commercial media bias. The tobacco industry is a major advertiser. According to the WHO (2013, p. 22), “the tobacco industry spends tens of billions of US dollars worldwide each year on tobacco advertising, promotion and sponsorship (TAPS). In the United States alone, the tobacco industry spends more than US$ 10 billion annually on TAPS activities”. The tobacco industry has suppressed reports on health risks of smoking, and induced media platforms to merely reprint statements claiming that there was no proven evidence for smoking inflicting health (Bagdikian 2004). For example, when the magazine Mother Jones published an article on smoking and health, the tobacco companies withdrew their ads and “made clear that Mother Jones would never get cigarette advertising again” (Whelan et al. 1981 p. 34). Warner and Goldenhar (1989) and Warner et al. (1992) provide strong statistical evidence that cigarette advertising in magazines relates to less coverage of the health risks of smoking.

News coverage of anthropogenic climate change is an issue of global importance. Global warming imperatively requires an accurately informed public. The broad scientific consensus is that human activities affect the climate (Oreskes 2004). The discourse in the news media, however, has significantly diverged from the scientific consensus, particularly in the US. Boykoff and Boykoff (2004) study the US press coverage of global warming between 1988 and 2002. They find that 53% of the investigated articles give equal weight to the scientific consensus opinion and the view that human activities are a negligible factor in overall changes in the climate. The difference between US television news coverage and the scientific consensus is even more severe: from 1996 to 2004, 70% of the television broadcasts on climate change provided a balanced view on its causes (Boykoff 2008, p. 6). As pointed out by Ellman and Germano (2009), one potential reason behind this biased media coverage is the influence of big advertisers such as car manufacturers or airlines.

Critical media reporting also has an important role to counteract misleading advertising. Again, tobacco is a case in point: cigarette advertising has often downplayed the associated health risks (see Glaeser and Ujhelyi 2010). Arguably, deceptive advertising, combined with tobacco advertisers’ influence on media content, explains why public awareness of the health risks lags decades behind their scientific discovery. For example, in Gallup polls from 1980 every second woman did not know that smoking during pregnancy increases the risk of stillbirth and miscarriage (Iscoe et al. 1980). Similarly, the WHO (2011) report on the global tobacco epidemic points out that many smokers do not fully understand the health risks of smoking.

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2See also Blasco and Sobbrio (2012) and Germano and Meier (2013).
3Sharp evidence on deceptive advertising is provided by Zinman and Zitzewitz (forthcoming), who show that ski resorts report 23% more fresh snow during weekends, when potential skiers are more flexible to react on snow conditions. Glaeser and Ujhelyi (2010) survey the available evidence. Nagler (1993) and Glaeser and Ujhelyi (2010) explore theoretical implications of misleading advertising, and study optimal policy responses. We show in an extension of our model in Section 3.1 of this Online-Appendix that misleading advertising strengthens the case for a cap on advertising.
The WHO (2011) also mentions that the news media are a key source of health information, and emphasizes the importance of media reporting on tobacco control.

Another well documented example is advertising for medical drugs. For example, Faerber and Kreling (2013) classify more than one half of all major claims in prescription and non-prescription drug ads on US television during 2008-2010 as potentially misleading. Furthermore, Faerber and Kreling (2013) report that consumers may see up to 30 hours of television drug advertising each year; in contrast, they spend 15 to 20 minutes at an average visit with their primary care physician. Misleading advertising may thus seriously impair consumers’ ability to take well informed decisions (Brody and Light 2011).

2 Microfoundations

A central assumption of our paper is that the willingness to pay of an advertiser for reaching a consumer decreases in the quality $v$ of the program the viewer watches. This section discusses the microfoundations mentioned in Section 4.3 of our paper.

One possible interpretation of the variable $v$ is that it corresponds to a genre preferred by viewers. The empirical results by Wilbur (2008) and Brown and Cavazos (2005) indicate that advertisers’ preferences over genres differ from viewers’ preferences: advertisers prefer lighter content. Similarly, experimental evidence from Goldberg and Gorn (1987) shows that happier program types put viewers in a more advertising receptive mood.

The implications for consumer welfare depend on the mechanism how program content impacts on advertising effectiveness. One potential reason is that television genres preferred by consumers are substitutes for consumption goods; therefore, the better the quality of the television program, the lower the willingness to pay for goods. In other words, viewing advertiser friendly genres is a complement for consumption, i.e., it raises the utility of the viewer from consuming goods. We call this the complementary microfoundation. Consumers have stable preferences defined over consumption and advertising. Television program quality enters their utility function directly, and it moreover affects the utility they gain from consumption goods. In particular, we assume that the gross utility gain of a consumer from buying a product of quality $\tilde{\sigma}$ is equal to $\tilde{\sigma} - \beta v$. If in addition producers capture all the gains on the product market, the willingness to pay of a producer of type $\tilde{\sigma}$ for an advertising slot is also $\tilde{\sigma} - \beta v$. In this microfoundation, consumers are rational, and the willingness to pay of consumers for goods indicates their true welfare gains from consumption.

Another reason why television content may have an impact on advertising effectiveness is that consumers’ recall of an ad depends on the program it is embedded in. Mathur and Chatterjee (1991) show in an experiment that viewers recall an ad better if it is shown in the context of a program that puts the viewers in a happy mood. This finding inspires our a second microfoundation, the recall microfoundation.

Suppose that some consumers recall an ad after seeing it, while others forget it. The probability that a consumer forgets an ad for a product of type $\tilde{\sigma}$ placed in a program of quality $v$ is $p(v, \tilde{\sigma})$. Plausibly, $p$ is increasing in $v$, and decreasing in $\tilde{\sigma}$: the better the television program, the more likely the viewer will recall an ad. This is the complementary view of advertising (Stigler and Becker 1977, Becker and Murphy 1993; for a survey see Bagwell 2007, esp. Section 2.4). While this view claims that advertising is a complement for the advertised consumption good, we claim here that television content may be both a complement and a substitute to consumption goods. We point out that our model of advertising itself is also consistent with the complementary view of advertising.
and the lower the product’s quality, the more likely the consumer is to forget the product. Moreover, recall of better products (i.e., those with a high $\bar{\sigma}$) might be less affected by television genre. A functional form consistent with these properties is $p(v, \bar{\sigma}) = \beta v / \bar{\sigma}$. A consumer who saw an ad for a product, but doesn’t recall it, does not buy the product; just as consumers who do not know the product exists. Consumers make rational decisions given their information. As in the complementary microfoundation, the willingness to pay of an informed consumer captures the true welfare gains from consumption. The willingness to pay of an advertiser of type $\bar{\sigma}$ for showing an ad to a consumer is $(1 - p(v, \bar{\sigma})) \bar{\sigma}$. Note this is decreasing in program quality $v$. Moreover, if $p(v, \bar{\sigma}) = \beta v / \bar{\sigma}$, the willingness to pay of the advertiser is $\bar{\sigma} - \beta v$, as in our model.

Television genre may also impact advertising effectiveness since it influences the moods of boundedly rational consumers. For an example, recall the case of Coca-Cola refusing to advertise during news broadcasts out of a concern that “bad” news might counteract its positioning of Coke as an “up-beat, fun product”. There is good empirical evidence that consumers’ moods impact their economic decisions. Harlé and Sanfey (2007) experimentally induce different moods by showing short movie clips to their subjects prior to an ultimatum game experiment. Incidental sad moods result in lower acceptance rates of unfair offers. Harlé et al. (2012) confirm this finding and study the underlying neural mechanisms in an fMRI study. Consumers have also been found in field data to be more likely to engage in impulse buying when they are in a positive mood (Beatty and Ferell 1998, Flight et al. 2012, see Faber and Fohs 2013 for a survey). Television induced moods may thus affect advertising effectiveness and purchase behavior, even when the “true” utility from consumption is not affected by television genre. We call this the moods microfoundation. In such a situation, consumers’ willingness to pay cannot simply be equated with their true utility gains from the products. If some genres put consumers in a spending happy mood such that they overestimate the true utility gains of the products, the welfare analysis has to take this into account; we do this in Section 3.1 of this Online-Appendix.

A second possible interpretation of the variable $v$ is that a high quality corresponds to more accurate and critical reporting over products, for example over any risks involved in the consumption. A program of higher quality can then be interpreted as containing more information that helps consumers making well-informed decisions. Good television programs may also contain information about the advertiser or producer, and any externalities that the products may have. An example is the case of government advertising and reporting of corruption scandals investigated by Di Tella and Francescelli (2011).

Following Nagler (1993) and Glaeser and Ujhelyi (2010), this interpretation can be used to develop another microfoundation, the deceptive advertising microfoundation. They argue that advertising sometimes is misleading and makes consumers underestimate costs involved in the consumption of their products. Key cases are advertising for medicines, cigarettes, and fast food; see also the discussion in Section 3.1 of this Online-Appendix. Following Glaeser and Ujhelyi (2010), suppose that consumption of a product has health costs $c$, so the true gain of consumers from consuming a product of type $\bar{\sigma}$ is $\bar{\sigma} - c$. Consumers’ perception of these costs may differ from the true costs. We assume that the perceived costs depend on how accurate reporting on television is; the better the program quality, the higher the perceived

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5Moreover, if producers capture all the rents on the product market as in our model, consumers have no incentive to remember the ads; forgetting is a form of rational ignorance.

6See also DellaVigna (2009) and Lerner et al. (2015) who survey the growing literature on the role of emotions in economic decisions.
Suppose that perceived costs are equal to \( c = \beta v_i \), and that, in the relevant range, consumers underestimate the costs, i.e., \( c > \beta v_i \). Thus, the better the program, the smaller consumers’ errors. Under these assumptions, a consumer is willing to pay up to \( \bar{\sigma} - \beta v_i \) for a product of quality \( \bar{\sigma} \). As above, we assume that the producer can completely capture these perceived benefits. Therefore, the willingness to pay of the producer for informing the consumer is equal to \( \bar{\sigma} - \beta v_i \), as well. Consumers are aware that a better program quality helps them making better decisions, and thus perceive a benefit \( v_i \) from watching the program. They are not aware that the products involve any health costs beyond \( \beta v_i \). Therefore, their perceived benefit from watching the broadcaster is

\[
w + v_i - \delta a_i - \tau x
\]

as in the paper. In the welfare analysis, however, we need to take into account that consumers do not correctly perceive the costs involved in the consumption decisions on the product market.

As discussed in the paper, the microfoundations described above are not mutually exclusive, and they all lead to the same positive predictions of the model. For normative questions, the main model in the paper builds on the complementarity microfoundation or the recall microfoundation, where consumers’ willingness to pay for a product accurately captures their true benefits from the product. The moods microfoundation and the deceptive advertising microfoundation, on the other hand, show that consumers may have losses on the product market since their perceived gains from the products are not equal to their true gains. The magnitude of these losses may depend both on advertising quantity and on television program quality. Section 3.1 of this Online-Appendix studies an extension of our main model that takes these considerations into account.

3 Extensions

3.1 Deceptive Advertising

In the paper we assume that a consumer’s willingness to pay for a product accurately captures the consumer’s benefits from the product. As argued above, this assumption is doubtful when purchase decisions are boundedly rational, or when advertising is suggestive or deceptive. Then, consumers may take suboptimal decisions (for themselves) on the product markets. Moreover, the corresponding losses of the consumers will depend on television program quality. This section investigates how taking these considerations into account modifies our main results.

As in the paper, we assume that a television program with quality \( v \) reduces the willingness to pay for a product of type \( \bar{\sigma} \) to \( \bar{\sigma} - \beta v \). However, here we assume that one part, \( \gamma \beta v \), of the reduction comes from consumers making smaller errors, and the remaining part \( (1 - \gamma) \beta v \) comes from good television being a substitute for consumption, where \( 0 \leq \gamma \leq 1 \). Following Glaeser and Ujhelyi (2010), suppose that consumption of a product has a health costs \( \gamma c \), so the true gain of consumers from buying a product of type \( \bar{\sigma} \) is \( \bar{\sigma} - (1 - \gamma) \beta v - \gamma c \). The consumer perceives the costs to be \( \gamma \beta v \); we assume that in the relevant range, consumers underestimate the costs, i.e., \( c > \beta v \). We scale both the true costs and the perceived costs with the same parameter \( \gamma \) in order to have one single parameter that captures the importance of deceptive.

\(^{7}\)Another difference between our setup and Glaeser and Ujhelyi (2010) is that, in our model, all consumers watching the same broadcaster are identical and have single unit demand. Therefore, even though each producer is a monopolist, consumption is efficient if and only if consumers perceive the health cost correctly, and there is no efficiency enhancing role for misinformation.
advertising. The case where $\gamma = 0$ corresponds to our main model. The case $\gamma = 1$ corresponds to the deceptive advertising microfoundation discussed in Section 2 of this Online-Appendix. When $0 < \gamma < 1$, higher television quality both reduces the true utility of consumption, and informs consumers so that they have a more accurate estimate of the costs.

As in the paper, we assume that the producers can capture all the perceived benefits from the products by charging the price $\tilde{\sigma} - \beta v$. The net utility gain of a consumer from consuming a good of type $\tilde{\sigma}$ is then $\tilde{\sigma} - (1 - \gamma) \beta v - \gamma c - (\tilde{\sigma} - \beta v) = \gamma (\beta v - c)$. The consumer is informed about, and consumes, in total $n$ such products, thus the consumer’s loss on the product market equals $a\gamma (c - \beta v)$. Consumers are aware that a better program quality helps to make better decisions, and perceive a benefit $v$ from watching the program. They are not aware that the products advertised on television involve any health costs beyond $\gamma \beta v$. Thus the consumers’ perceived benefit from watching a broadcaster is given by (1), and hence as in the paper. Consumer surplus, however, also has to take into account consumers’ losses on the product market:

$$CS = n (w + v - \delta a) - \frac{\eta \tau}{4N} - n a\gamma (c - \beta v). \quad (2)$$

Producer surplus is, as in the paper, given by

$$PS = n \int_0^a \left( \sigma - \beta v - \frac{\sigma x}{m} \right) dx. \quad (3)$$

Note that consumption of a product of type $\tilde{\sigma}$ raises welfare if and only if $\tilde{\sigma} > \gamma c$. Thus consumption of high-quality goods is welfare enhancing in our setting.

For the positive analysis, this model generates the same predictions as the model in the paper. A cap on advertising, however, now has additional benefits for the consumers: it improves their decisions on the product market, both by reducing the number of ads and by improving the program quality. Thus, the welfare gains due to a cap are higher than in the paper. To see this formally, note that the effects of lower advertising quantity, and higher program quality, on consumer surplus are

$$-\frac{\partial CS}{\partial a} = \delta n + n\gamma (c - \beta v),$$

$$-\frac{\partial CS}{\partial v} = n + na\gamma \beta.$$ 

Therefore, both the direct (less advertising) and the indirect (higher program quality) effect of a cap on consumer surplus are more important when $\gamma > 0$. It is therefore more likely that the cap’s positive effects on consumer surplus outweigh the negative effects on producer surplus.

Deceptive advertising thus makes the case for a cap stronger. It modifies, however, our result on the complementarity between competition and regulation. When there are many independent broadcasters, consumers’ errors are small since program quality is high and advertising quantities are low; thereby consumers’ gains from a cap are smaller. This works against the complementarity between competition and regulation. Indeed, if $\gamma$ is close to 1, competition and regulation are no longer local complements.

We now show, however, that the local complementarity between competition and regulation holds whenever $\gamma < (2\sigma + m\beta \delta) / (3\sigma + m\beta \delta)$; a sufficient condition is $\gamma < 2/3$. Competition and regulation are local complements if the marginal welfare gains from a cap, $-\frac{\partial W}{\partial a}$, are increasing in $N$. Here, $W = CS + PS - NF$, where $CS$ is given in (2), and $PS$ in (3). Substituting
from Lemma 1 of the paper into these expressions, and differentiating, one obtains

$$\frac{\partial}{\partial N} \left( -\frac{dW}{d\tilde{a}} \right) = n\beta\tau \frac{2\sigma - 3\sigma \gamma + m\beta \delta - m\beta \gamma \delta}{N^2 (\sigma + m\beta \delta)},$$

which is strictly positive if and only if $\gamma < \left( 2\sigma + m\beta \delta \right) / \left( 3\sigma + m\beta \delta \right)$.

### 3.2 Sector Specific Regulation

In the paper we assumed that advertisers have a shared interest in low program quality. This may be appropriate for specific industries where the qualities of the products sold are highly correlated. For example, all producers in the tobacco industry may suffer from an increased awareness of the health risks of smoking. In other industries, however, broadcasters may be less hostile to accurate reporting. As argued by Ellman and Germano (2009) and Germano and Meier (2013), and modelled in detail by Blasco et al. (2014) and Spiteri (2015), competition on the product market can ameliorate commercial media bias when advertisers have opposing interests.

In this section, we study an extension where advertisers are interested in low program quality in some industries, but not in others. This allows us to probe the robustness of our results, and to shed light on the rationale of sector specific bans on television advertising. For example, in the United States, broadcasters are not allowed to send commercials on tobacco by the Public Health Cigarette Smoking Act of 1970 (Tobacco Control and Legal Consortium 2012). The rules in the European Union are similar. The Audiovisual Media Services Direction (2010) bans commercials on cigarettes and other tobacco products, medicinal products and medicinal treatment available only on prescription. The advertisements for alcoholic beverages shall not be aimed specifically at minors and shall not encourage immoderate consumption (Article 9). The restrictions can be more stringent in the member states. In Germany, gambling must not be advertised and commercials for alcohol must not appeal to children or teenagers (Seufert and Gundlach 2012).

We give a new rationale for sector specific regulations based on their impact on non-advertising media content below. To keep the discussion short, we focus on the case where $\delta = 0$. Suppose there is a mass $m_1$ of type 1 advertisers characterized by $\beta = 0$. These advertisers are not interested in dumbing down media content. Moreover, there is a mass $m_2 = m - m_1$ of type 2 advertisers with $\beta > 0$; these advertisers prefer lower program quality. Suppose that the quality $\tilde{\sigma}$ of the product of any advertiser is drawn from the uniform distribution on $[0, \sigma]$; thus type 1 and type 2 advertisers do not differ in this respect. (In Section 4.2 we explore an extension where advertisers’ preferences for media content depend on their product’s quality). Advertising demand of broadcaster $i$ is then

$$a_i = m_1 \Pr (\tilde{\sigma} > r_i) + m_2 \Pr (\tilde{\sigma} - \beta v_i > r_i)$$

$$= \begin{cases} 
  m_1 \left( 1 - \frac{r_i}{\sigma} \right) + m_2 \left( 1 - \frac{r_i + \beta v_i}{\sigma} \right), & \text{if } 0 \leq r_i < \sigma - \beta v_i, \\
  m_1 \left( 1 - \frac{r_i}{\sigma} \right), & \text{if } \sigma - \beta v_i \leq r_i < \sigma.
\end{cases}$$

Inverse ad demand per viewer is

$$r_i = \begin{cases} 
  \sigma - \sigma \frac{a_i}{m_1}, & \text{if } a_i < \frac{m_1 \beta v_i}{\sigma}, \\
  \sigma - \frac{m_2 \beta}{m} v_i - \frac{a_i}{m}, & \text{if } \frac{m_1 \beta v_i}{\sigma} \leq a_i < \frac{1}{\sigma} (m \sigma - \beta m_2 v_i).
\end{cases}$$
If $m_1$ is sufficiently big, then only the type 1 advertisers are advertising in equilibrium, since the willingness to pay of type 2 advertisers is lower. In this case the market solves the problem of commercial media bias. If $m_1$ is sufficiently small, however, there exists a symmetric equilibrium where both type-1 and type-2 advertisers are served.\footnote{If the maximum quality $\bar{v}$ is sufficiently high, there also exist an equilibrium where all broadcasters choose very high quality and sell advertising spots only to type-1 producers. Any broadcaster trying to sell to type-2 producers would have to lower its quality so much that it would not have any viewers.} In this case, the profit of broadcaster $i$ is

$$\pi_i = n \left( \frac{1}{N} + \frac{v_i - \delta a_i - u}{\tau} \right) \left( \sigma - \beta_2 v_i - \frac{\sigma a_i}{m} \right) a_i$$

where $\beta_2 := m_2 \beta / m < \beta$. The equilibrium values of program quality and advertising quantity can be found by replacing $\beta$ by $\beta_2$ in the formulas in Proposition 1 and Lemma 1 in the paper.\footnote{The proof is similar to the proofs of these results, with one additional consideration: one has to take into consideration deviations to a high quality, whereby the deviating broadcaster captures all viewers, and serves only type-1 advertisers. The profits from this deviation are proportional to $m_1$; the deviation does not pay of $m_1$ is sufficiently small.}

Consumer surplus can be calculated as in the paper (see equation (5) there). To calculate producer surplus, we need to take the two different types of advertisers into account:

$$PS = n \int_0^{\frac{m_1 \beta v}{\sigma}} \left( \sigma - \frac{\sigma x}{m_1} \right) dx + n \int_{\frac{m_1 \beta v}{\sigma}}^{a} \left( \sigma - \frac{m_2 \beta v}{m} - \frac{\sigma x}{m} \right) dx.$$

We now consider the effect of a general cap that applies to the quantity of all advertising by type 1 and type 2 producers.

**Proposition 1** Consider the extension where there is a mass $m_1$ of advertisers with $\beta = 0$ and a mass $m_2 = m - m_1$ of advertisers with $\beta > 0$. Let $\delta = 0$. If both types of advertisers are served in equilibrium, a local cap on advertising increases welfare if and only if

$$N > \hat{N}_{\text{cap}}^2 := \frac{2\beta^2 \tau m_2}{\sigma (\beta (m - m_2) + 1)}.$$

**Proof.** Since by assumption $\delta = 0$,

$$W = nw + nv + n \int_0^{\frac{m_1 \beta v}{\sigma}} \left( \sigma - \frac{\sigma x}{m_1} \right) dx + n \int_{\frac{m_1 \beta v}{\sigma}}^{a} \left( \sigma - \frac{m_2 \beta v}{m} - \frac{\sigma x}{m} \right) dx - \frac{nt}{4N}.$$

The effect of a cap on welfare is

$$\frac{dW}{da} = \frac{\partial W}{\partial a} + \frac{dv}{da} \frac{\partial W}{\partial v},$$

where

$$\frac{\partial W}{\partial a} = n \left( \sigma - \frac{m_2 \beta v}{m} - \frac{\sigma a}{m} \right),$$

$$\frac{\partial W}{\partial v} = n - n \frac{m_2 \beta}{m} \left( a - \frac{m_1 \beta v}{\sigma} \right).$$
Moreover, from Lemma 1, replacing $\beta$ by $\beta_2$, we have

$$\frac{dv}{da} = -\frac{1}{m} \frac{\sigma}{\beta_2} = -\frac{\sigma}{\beta m_2}.$$ 

Thus

$$\frac{dW}{da} = n \left( \frac{\sigma - m_2 \beta}{m} \frac{v}{m} - \frac{\sigma a}{m} \right) - \frac{\sigma}{\beta m_2} \left( n - n \frac{m_2 \beta}{m} \frac{a - m_1 \beta v}{\sigma} \right).$$

The equilibrium values of $a$ and $v$ can be taken from Proposition 1 in the paper, replacing $\beta$ by $\beta_2$ and setting $\delta = 0$:

$$a = \frac{m \beta_2 \tau}{N \sigma} = \frac{\beta m_2}{N \sigma},$$
$$v = \frac{\sigma}{\beta_2} - \frac{2 \tau}{N} = \frac{m \sigma}{m_2 \beta} - \frac{2 \tau}{N}.$$

Inserting these into equation (4) shows that the effect of a local cap is

$$\frac{dW}{da} = \frac{n \beta m_2}{Nm} - \frac{\sigma}{\beta m_2} \left( n - n \frac{m_2 \beta}{m} \left( \frac{\beta m_2}{N \sigma} - \frac{m_1 \beta}{\sigma} \left( \frac{m \sigma}{m_2 \beta} - \frac{2 \tau}{N} \right) \right) \right)$$
$$= \frac{n \beta m_2}{Nm} - \frac{\sigma}{m_2 \beta} \left( n + n \beta \left( m_1 \beta - \frac{\beta \tau m_2^2 + 2 m_1 m_2}{N m \sigma} \right) \right).$$

This is strictly negative if and only if

$$\frac{\beta m_2}{N m} < \frac{\sigma}{m_2 \beta} \left( 1 + \beta \left( m_1 \beta - \frac{\beta \tau m_2^2 + 2 m_1 m_2}{N m \sigma} \right) \right).$$

or, equivalently (since $m = m_1 + m_2$), $N > \bar{N}_{\text{cap}}$.

To compare this with our main model, first note that when $m_2 \to m$, we get the same condition as in Proposition 4 in the paper for the case $\delta = 0$. Moreover, $\bar{N}_{\text{cap}}$ is increasing in $m_2$. Thus the lower $m_2$, the more likely it is that a cap improves welfare. Therefore, in the extension considered in Proposition 1 of the Online-Appendix, it is more likely than in our main model that a cap improves welfare. The intuition is that, since only some broadcasters suffer from higher program quality, the loss of producer surplus due to a cap is not as important as in the main model in the paper.

As reported above, many countries impose bans on advertising for specific sectors or products, for example tobacco or alcohol. To see the implications in our model, consider a sector specific advertising ban that excludes all type 2 advertisers. Then for the broadcasters there is no drawback from choosing high program quality; thus in equilibrium program quality will be equal to its highest possible level $\bar{v}$. If $\bar{v}$ is sufficiently high, a sector specific advertising ban leads to a higher welfare than laissez faire, or a local cap on all advertising. While most rationales for regulating the content of advertising are built on bounded consumer rationality, this argument identifies conditions such that regulating advertising content is justifiable for the reason it decreases commercial media bias, even when consumers are perfectly rational.

### 4 Robustness

The following section contains robustness checks with respect concerning television viewing behavior and advertising demand.
4.1 Television viewing behavior

This section probes the robustness of our results with respect to the model of television viewing behavior, focussing on the case where the number of broadcasters is exogenous. One limitation of our results comes from the assumption that everyone watches television. If the number of viewers is endogenous, and viewers are ad averse, a cap on advertising will ceteris paribus increase the total number of viewers. This means that increasing $v$ has higher costs for a broadcaster, because he loses $\beta a$ on every viewer he has, countervailing the quality improving effects of a cap.\(^{10}\)

Our results do not hinge, however, on specific features of the Salop circle model. To show this, we introduce a more general model of television viewing behavior that nests the Salop model and several other textbook models of discrete choice. Suppose that, if all broadcasters $j \neq i$ behave symmetrically, the fraction of viewers who watch broadcaster $i$ depends only on the difference between the utility $v_i - \delta a_i$ offered by broadcaster $i$, and the utility offered by the competitors, scaled by a factor $1/\tau$. The share is given by $s\left(\frac{v_i - \delta a_i - u}{\tau}\right)$, where $u := v_j - \delta a_j$, and $s$ is a strictly increasing function with $s(0) = 1/N$. In general, the function $s$ will depend on $N$; we assume it to be independent of the other exogenous parameters of the model. We assume that the function $s$ is sufficiently well behaved such that a symmetric equilibrium in pure strategies exists and can be characterized by the relevant first order conditions. This model nests the Salop model with linear transportation costs studied in the paper (given undercutting is not an issue), the Salop model with any convex (e.g. quadratic) transportation costs, the Logit model (see, for example, Anderson, de Palma, and Thisse 1992), and the covered Spokes model introduced by Chen and Riordan (2007) and used in a study of commercial media bias by Germano and Meier (2013).

Sections 4.3.1 to 4.3.3 below substantiate these claims, and Section 4.3.4 characterizes the symmetric equilibrium with and without a cap. Here we summarize the main results. A cap increases program quality, and indeed $dv/(d\bar{a}) = -\sigma/\beta m$ exactly as in the paper, see equation (13) there. The comparative static of the equilibrium depends on the behavior of $Ns'(0)$. The advertising quantity $a$ is decreasing in $N$, and quality $v$ is increasing in $N$, if and only if $Ns'(0)$ is increasing in $N$. Similarly, the price of an advertising spot per viewer $r$ is decreasing in $N$ if and only if $Ns'(0)$ is increasing in $N$. As discussed in the paper, this seems to be the empirically plausible case. The Salop model with linear or strictly convex transportation costs, and the Logit model share this property that $Ns'(0)$ is increasing in $N$. The Spokes model is a limit case where $Ns'(0)$ is independent of $N$.

For the welfare analysis, we assume that, for a given number of broadcasters $N$, welfare is given by

$$W = n \left( w + v - \delta a \right) + n \int_0^a \left( \sigma - \beta v - \frac{\sigma}{m} x \right) dx - NF + C(N),$$

where $C(N)$ is independent of $a$ and $v$. This is the case in all the discrete choice models mentioned above (e.g. in the Salop model, $C(N)$ equals aggregate transportation costs). As in the paper, a cap increases consumer surplus, and decreases producer surplus. Moreover, the welfare analysis of a local cap is quite similar to our main model. A local cap improves welfare if and only if $Ns'(0) > N_{cap}$. This is, in addition, a sufficient but not necessary condition for the

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\(^{10}\)Indeed, if ad aversion is strong, a cap may decrease equilibrium program quality. The simplest way to see this is to reconsider the example from Section 3 of the paper, and to introduce ad aversion $\delta > 0$. Then a cap will increase program quality whenever $\delta < 1$, but it will decrease quality when $\delta > 1$.\n

optimal cap subject to the constraint that profits are nonnegative to be binding. The optimal policy is either to choose a cap that drives profits down to zero, i.e., $\bar{a} = FN^2 s'(0) / (n \beta \tau)$, or laissez-faire.

As seen above, many standard discrete choice models imply that $Ns'(0)$ is increasing in $N$. Moreover, if $Ns'(0)$ is increasing in $N$, then the comparative statics of equilibrium advertising quantity, program quality, and price per ad per viewer go in empirically plausible directions. It is exactly this property that also gives rise to the complementarity between competition and regulation: If $Ns'(0)$ is increasing in $N$, then an increase in $N$ makes it more likely that a local cap raises welfare.

Since $s(\cdot)$ is independent of the remaining parameters of the model, their impact is exactly as in the linear Salop model considered in the paper. Table 2 lists the market share, the condition under which a local cap improves welfare, and the optimal cap (if binding) for several discrete choice models nested in our general model. It shows that the conditions under which a local cap improves welfare are qualitatively similar. Moreover, the optimal cap has the same qualitative properties under these models. Table 2 also shows, however, that the precise quantitative implications depend on the assumed model of television viewing. The condition under which a local cap improves welfare is the most restrictive in the Logit model, followed in decreasing strength by the Spokes model, the Salop model with linear transportation costs and the Salop model with quadratic transportation costs.

<table>
<thead>
<tr>
<th>Model</th>
<th>$s \left( \frac{u_i - u_j}{\tau} \right)$</th>
<th>$Ns'(0)$</th>
<th>local cap improves $W$ iff</th>
<th>zero-profit cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salop linear</td>
<td>$\frac{1}{N} + \frac{u_i - u_j}{\tau}$</td>
<td>$N$</td>
<td>$N &gt; \bar{N}_{cap}$</td>
<td>$\frac{FN^2}{n \beta \tau}$</td>
</tr>
<tr>
<td>Salop quadratic</td>
<td>$\frac{1}{N} + \frac{N(u_i - u_j)}{\tau}$</td>
<td>$N^2$</td>
<td>$N^2 &gt; \bar{N}_{cap}$</td>
<td>$\frac{FN^2}{n \beta \tau}$</td>
</tr>
<tr>
<td>Spokes</td>
<td>$\frac{1}{N} + \frac{u_i - u_j}{N \tau}$</td>
<td>$1$</td>
<td>$1 &gt; \bar{N}_{cap}$</td>
<td>$\frac{FN}{n \beta \tau}$</td>
</tr>
<tr>
<td>Logit</td>
<td>$\frac{e^{u_i / \tau}}{e^{u_i / \tau} + (N - 1)e^{u_j / \tau}}$</td>
<td>$\frac{N - 1}{N}$</td>
<td>$\frac{N - 1}{N} &gt; \bar{N}_{cap}$</td>
<td>$\frac{F(N - 1)}{n \beta \tau}$</td>
</tr>
</tbody>
</table>

Table 2: Different assumptions on television viewing behavior. Salop linear (quadratic) refers to the Salop model with linear (quadratic) transportation costs.

### 4.2 Advertising demand

In our paper we assume that a higher program quality reduces the willingness to pay of all advertisers by the same amount. In a diagram with advertising quantity on the horizontal axis, and the price of an ad per viewer on the vertical axis, the inverse demand curve for advertising spots is linear, and a higher program quality leads to a parallel downward shift of the inverse demand curve; see the left hand side of Figure 1 of this Online-Appendix. The reduction of the willingness to pay, however, may depend on the quality of the good. It seems plausible to assume that the willingness to pay of producers of high (rather than low) quality goods is less affected by program quality. Moreover, there might be nonlinearities in the inverse advertising demand curve. For example, advertisers may have increasing marginal costs from program quality, or similarly, viewers' marginal utility from program quality may be decreasing.

To study potential implications, suppose that the inverse demand for advertising is given by a function $r(a, v)$, which is decreasing in $a$ and in $v$. While a complete analysis is beyond the

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11 Note that even in the Spokes model a lower $\tau$, i.e., a more competitive broadcasting market since programs are better substitutes, makes a cap more likely to be welfare enhancing.

12 Note this can also capture decreasing marginal utility of the viewers. Suppose a viewer's utility is given by
scope of this Online-Appendix, we show that the plausible assumption that producers of high quality goods have little to lose from program quality strengthens the mechanism by which a cap increases program quality. Formally, the assumption means that the cross-partial derivative \( r_{va} := \frac{\partial^2 r}{\partial v \partial a} \) is negative. In terms of the right hand side of Figure 1, an increase in quality makes the inverse ad demand curve steeper.

Consider the profit maximization problem of broadcaster \( i \), given a binding cap \( \hat{a} \):

\[
\max_{v_i} ns \left( \frac{v_i - \delta \hat{a} - u}{\tau} \right) r(\hat{a}, v_i) \hat{a} - F
\]

where \( u = v_j - \delta \hat{a} \) for all \( j \neq i \). Setting up the first order condition for the profit maximizing program quality, and then using symmetry, one obtains

\[
\frac{s'(0)}{\tau} r(\hat{a}, v) + \frac{1}{N} r_v(\hat{a}, v) = 0
\]

where \( r_v \) is the partial derivative \( \partial r/\partial v \). Consider how the equilibrium value of \( v \) changes when the cap \( \hat{a} \) changes. From the implicit function rule,

\[
\frac{dv}{d\hat{a}} = -\frac{s'(0) r_a + \frac{1}{N} r_{va}}{s'(0) r_v + \frac{1}{N} r_{vv}}
\]

where as above subscripts indicate partial derivatives. The denominator is negative by the second order condition, and \( r_a < 0 < s'(0) \) by assumption. Therefore, if \( r_{va} = 0 \), then \( dv/d\hat{a} < 0 \), and a cap improves program quality as in the paper; the sign of the second order partial derivatives \( r_{vv} \) and \( r_{aa} \) does not matter for this result.\(^{13}\) As can be seen from (5), \( r_{va} < 0 \) reinforces the quality enhancing effect of the cap.

The economics behind this is as follows. A tighter cap implies that the marginal advertiser has a better product. When \( r_{va} < 0 \), it follows that the willingness to pay of the marginal advertiser decreases less in program quality. Therefore, broadcasters have an additional reason to increase their program quality. Similarly, any reason why the marginal advertiser has a lower stake in program quality works in the same direction. On the other hand, when the marginal advertiser has a higher stake in program quality (perhaps because of the advertisers’ cost structure), this works against the quality enhancing effect of a cap.

\[w + f(v_i) - \delta a_i - \tau x, \text{ where } f'(v) > 0 > f''(v), \text{ instead of equation (1), and } r_i \text{ is as given by } r_i = \sigma - \beta v_i - \frac{a_i}{m}.\]

Then we can equivalently think of the broadcaster choosing \( \tilde{v}_i := f(v_i) \); then viewers’ utility from watching is \( w + \tilde{v}_i - \delta a_i - \tau x \) as in (1), but \( r_i = \sigma - \beta f^{-1}(\tilde{v}_i) - \sigma a_i/m \), thus the advertisers have increasing marginal costs from program quality.

\(^{13}\)While the curvature of \( r \) in \( v \) does not matter for the sign of \( dv/d\hat{a} \), it influences its absolute value. This can change the results on the desirability of a cap and on the local complementarity between competition and regulation. To see this, consider the case where there is some upper bound \( \bar{v} \) on quality. Such an upper bound might endogenously arise as the result of viewers’ utility being increasing in \( v \) only up to \( \bar{v} \), or from advertisers’ willingness to pay dropping rapidly when quality exceeds \( \bar{v} \). Then, enough competition on the media market can be sufficient to ensure that equilibrium program quality is \( \bar{v} \), and thus as high as it possibly can be. If, in addition, ad aversion is small, a cap on advertising will be detrimental to welfare.
Figure 1: Left hand side: In the main model, higher program quality \( v \) implies a parallel downwards shift of the inverse ad demand function. Right hand side: When producers of high quality goods are less affected by program quality, the inverse ad demand function becomes steeper when \( v \) increases.

4.3 Technical Details

4.3.1 Salop model with convex transportation costs

Suppose that the transportation costs of a viewer located at a distance \( x \) from the broadcaster equals \( l(x) \), where \( l \) is strictly increasing, strictly convex, and satisfies \( l(0) = 0 \). Let \( u_i = v_i - \delta a_i \), and \( u = v_j - \delta a_j \) for all \( j \neq i \). Assuming that there is an indifferent viewer between broadcaster \( i \) and its closest competitors, the distance \( x \) between this viewer and broadcaster \( i \) solves

\[
 u_i - l(x) = u - \frac{1}{N} - x,
\]

or equivalently

\[
 \frac{u_i - u}{\tau} = l(x) - \frac{1}{N} - x.
\]

Let \( \lambda(x, N) = l(x) - l\left(\frac{1}{N} - x\right) \). Since \( \lambda \) is strictly increasing in \( x \), holding \( N \) fixed, an inverse function \( \lambda^{-1}(\cdot, N) \) exists, and

\[
 \lambda^{-1}\left(\frac{u_i - u}{\tau}, N\right) = x.
\]

Therefore, the market share of \( i \) is

\[
 s\left(\frac{u_i - u}{\tau}\right) = 2x = 2\lambda^{-1}\left(\frac{u_i - u}{\tau}, N\right).
\]

Thus

\[
 s'\left(\frac{u_i - u}{\tau}\right) = \frac{2}{\lambda'(\lambda^{-1}(\frac{u_i - u}{\tau}, N))} = \frac{2}{l'(x) + l'\left(\frac{1}{N} - x\right)}.
\]

When \( u_i - u = 0 \), \( x = 1/(2N) \). Thus

\[
 Ns'(0) = \frac{N}{l'\left(\frac{1}{2N}\right)}.
\]

Therefore

\[
 \frac{d}{dN} (Ns'(0)) = \frac{l'\left(\frac{1}{2N}\right) + l''\left(\frac{1}{2N}\right)\frac{1}{2N}}{l'\left(\frac{1}{2N}\right)^2} > 0.
\]

Thus \( Ns'(0) \) is increasing in \( N \).

It remains to establish that the first order conditions are sufficient for a maximum. Given the convex transportation costs, undercutting is not an issue. To show that any critical point is a global maximum, we now show that the variable profit, \( \pi_i + F \), is log-concave in \( (a_i, v_i) \).

We begin by establishing that \( \ln s\left(\frac{u_i - u}{\tau}\right) \) is strictly concave in \( u_i \).

\[
 \frac{\partial^2}{\partial u_i^2} \left( \ln s\left(\frac{u_i - u}{\tau}\right) \right) = \frac{1}{\tau^2 s\left(\frac{u_i - u}{\tau}\right)^2} \left( s\left(\frac{u_i - u}{\tau}\right) s''\left(\frac{u_i - u}{\tau}\right) - s'\left(\frac{u_i - u}{\tau}\right)^2 \right).
\]
is strictly smaller zero if, and only if,

\[ s \left( \frac{u_i - u}{\tau} \right) s'' \left( \frac{u_i - u}{\tau} \right) < \left( s' \left( \frac{u_i - u}{\tau} \right) \right)^2. \]

Here, this inequality is equivalent to

\[ 1 + \lambda^{-1} \left( \frac{u_i - u}{\tau}, N \right) \frac{\lambda'' \left( \lambda^{-1} \left( \frac{u_i - u}{\tau}, N \right) \right)}{\lambda' \left( \lambda^{-1} \left( \frac{u_i - u}{\tau}, N \right) \right)} > 0 \]

which is true since \( \lambda^{-1} > 0, \lambda' > 0 \) and \( \lambda'' > 0 \).

It follows that

\[ \ln (\pi_i + F) = \ln s \left( \frac{u_i - u}{\tau} \right) + \ln \left( \sigma - \beta v_i - \frac{\sigma a_i}{m} \right) + \ln a_i \]

is the sum of three functions, each of them being weakly concave in \((a_i, v_i)\). Moreover, the first of these functions is strictly concave in \((a_i, v_i)\) except along the line where \( v_i - \delta a_i \) is constant; the second is strictly concave except along the line where \( \beta v_i + \frac{\sigma a_i}{m} \) is constant. It follows that \( \ln (\pi_i + F) \) is strictly concave in \((a_i, v_i)\).

**4.3.2 Logit**

In the logit model, the market share of broadcaster \( i \) is

\[ s \left( \frac{u_i - u}{\tau} \right) = \frac{e^{u_i/\tau}}{e^{u_i/\tau} + (N - 1)e^{u_i/\tau}} \]

(see, for example, Anderson, de Palma, and Thisse 1992). It is straightforward to calculate that

\[ Ns'(0) = \frac{N - 1}{N}. \]

Moreover, it can be shown that the variable profit \( (\pi_i + F) \) is log-concave in \((a_i, v_i)\). Therefore, the first order conditions are sufficient for a maximum.

**4.3.3 Spokes**

In the covered Spokes model introduced by Chen and Riordan (2007) and used in Germano and Meier (2013),

\[ s_i \left( \frac{u_i - u}{\tau} \right) = \frac{1}{N} + \frac{1}{N} \frac{u_i - u}{\tau}. \]

Then \( Ns'(0) = 1 \) is independent of \( N \). Again, the variable profit is log-concave in \((a_i, v_i)\) and therefore the first order conditions are sufficient for a maximum.

**4.3.4 Equilibrium characterization and welfare effects of a cap**

The profit of broadcaster \( i \) is

\[ \pi_i = ns \left( \frac{v_i - \delta a_i - u}{\tau} \right) \left( \sigma - \beta v_i - \frac{\sigma a_i}{m} \right) a_i - F. \]
The first order condition
\[
\frac{\partial \pi_i}{\partial v_i} = s' \left( \frac{v_i - \delta a_i - u}{\tau} \right) \frac{1}{\tau} v_i \left( \sigma - \beta v_i - \frac{\sigma}{m} a_i \right) a_i - \beta n s \left( \frac{v_i - \delta a_i - u}{\tau} \right) a_i = 0
\]
simplifies to, assuming symmetry,
\[
r \equiv \sigma - \beta v - \frac{\sigma}{m} a = \frac{\beta \tau}{N s'(0)}.
\]
Consider the case without a cap. The first order condition
\[
\frac{\partial}{\partial a_i} \pi_i = ns' \left( \frac{v_i - \delta a_i - u}{\tau} \right) \left( -\frac{\delta}{\tau} \right) \left( \sigma - \beta v_i - \frac{\sigma}{m} a_i \right) a_i + ns \left( \frac{v_i - \delta a_i - u}{\tau} \right) \left( -\frac{\sigma}{m} \right) a_i
\]
\[
+ n s \left( \frac{v_i - \delta a_i - u}{\tau} \right) \left( \sigma - \beta v_i - \frac{\sigma}{m} a_i \right)
\]
\[
= 0
\]
simplifies, in any symmetric equilibrium, to
\[
-\frac{\delta}{\tau} s'(0) r a - \frac{1}{N} \frac{\sigma}{m} a + \frac{r}{N} = 0.
\]
Inserting \( r = \beta \tau / (N s'(0)) \) and solving for \( a \) gives
\[
a = \frac{\beta}{N s'(0)} \frac{m \tau}{\sigma + m \beta \delta}.
\]
Moreover,
\[
v = \frac{\sigma}{\beta} - \frac{\tau}{N s'(0)} \frac{2\sigma + m \beta \delta}{\sigma + m \beta \delta}.
\]
If there is a binding cap \( \bar{a} \), then in any symmetric equilibrium
\[
v = \frac{\sigma}{\beta} - \frac{\sigma}{\beta m} \bar{a} - \frac{\tau}{N s'(0)}.
\]
Inserting the equilibrium value of \( v \) into the welfare function, we get
\[
W(\bar{a}) = n \left( w + \frac{\sigma}{\beta} - \frac{\sigma}{\beta m} \bar{a} - \frac{\tau}{N s'(0)} - \delta \bar{a} \right) + n \int_{0}^{\bar{a}} \left( \beta \left( \frac{\sigma m \bar{a}}{N s'(0)} + \frac{\tau}{N s'(0)} \right) - \frac{\sigma}{m} x \right) dx - NF + C
\]
\[
= n \left( -\frac{\sigma}{\beta m} \bar{a} - \delta \bar{a} \right) + \frac{n \sigma}{2m} \bar{a}^{2} + \frac{n \beta \bar{a} \tau}{N s'(0)} + \text{terms independent of } \bar{a}.
\]
Consider the problem to maximize \( W(\bar{a}) \) by choosing \( a \), subject to the constraint that profits are nonnegative:
\[
\pi_i = \frac{n \beta}{N N s'(0)} \bar{a} - F \geq 0.
\]
Since \( W(\bar{a}) \) is convex in \( \bar{a} \), either the optimal cap is driving profits to zero, or it is not binding. Moreover,
\[
\frac{dW}{d\bar{a}} = n \left( -\frac{\sigma}{\beta m} - \delta \right) + \frac{\bar{a}}{m} + \frac{n \beta \tau}{N s'(0)}.
\]
Evaluating this at the equilibrium level of \( a \) (absent a cap) gives

\[
\frac{dW}{da} = n \left( -\frac{\sigma}{\beta m - \delta} \right) + \frac{n^\beta N^{- \alpha} \frac{m^\tau}{\alpha + m \beta \delta}}{\sigma + m \beta \delta} \frac{1}{N s'(0)} + \frac{n \beta \tau}{N s'(0)}
\]

Rearranging shows that this is strictly negative if and only if

\[
N s'(0) > \frac{(2 \sigma + m \beta \delta) m^2 \tau}{(\sigma + m \beta \delta)^2} = \hat{N}_{\text{cap}}.
\]

Thus a local cap raises welfare if and only if \( N s'(0) > \hat{N}_{\text{cap}} \).

5 Omitted Steps of Proof of Proposition 1

In Appendix A.1, we have shown that if a symmetric equilibrium exists, it is as described in Proposition 1 in the paper. Here we prove existence of the equilibrium. We suppose all broadcasters \( j \neq i \) behave as indicated in the Proposition and show that broadcaster \( i \) has no incentive to deviate. Since the proof is somewhat lengthy, we break it down into steps, for which we first briefly sketch the intuition. \textit{Step 1} assumes that broadcaster \( i \) does not undercut and shows that, under this assumption, broadcaster \( i \) has no incentive to deviate. The remaining steps consider deviations that involve undercutting. \textit{Step 2} prepares the ground by describing the range of \( a_i \) and \( v_i \) leading to undercutting. From here, it is straightforward to show that undercutting more than two rivals is not profitable: it leads to zero inverse ad demand and hence to a profit of zero (\textit{Step 3}). The remaining steps consider undercutting two rivals. \textit{Step 4} considers the case of \( N = 3 \). Here, by undercutting two rivals, broadcaster \( i \) captures all the market. It will choose \( v_i \) such that it just undercut its two rivals since this is sufficient to make all viewers watch broadcaster \( i \). We show that the resulting profit is smaller than the equilibrium profit. \textit{Steps 5} and \( 6 \) consider undercutting two rivals in case of \( N > 3 \). Here, by undercutting two rivals, broadcaster \( i \) does not capture all viewers. \textit{Step 5} shows that broadcaster \( i \) will not increase its program quality more than necessary to just undercut two rivals. The intuition is that at the equilibrium broadcaster \( i \) is already indifferent whether or not to increase its program quality a bit, thereby winning viewers but gaining a lower price for ads. At the considered deviation, broadcaster \( i \) already has more viewers than in equilibrium, and thus prefers not to increase its program quality any more. \textit{Step 6} shows that the deviation profit from just undercutting two rivals is smaller than the equilibrium profit.

\textit{Step 1: Find the profit maximizing decisions of broadcaster \( i \) assuming that broadcaster \( i \) does not undercut any rival.}

Suppose all broadcasters \( j \neq i \) behave as indicated. Then

\[
u = v_j - \delta a_j = \frac{\sigma}{\beta} - \frac{2 \tau}{N}.
\]

The profit of broadcaster \( i \) is

\[
\pi_i = n \left( \frac{1}{N} + \frac{v_i - \delta a_i - \left( \frac{\sigma}{\beta} - \frac{2 \tau}{N} \right)}{\tau} \right) \left( \sigma - \beta v_i - \frac{\sigma a_i}{m} \right) a_i
\]
whenever
\[
\left(1 - \frac{a_i}{m}\right) \frac{\sigma}{\beta} > v_i > \delta a_i + \left(\frac{\sigma}{\beta} - \frac{2\tau}{N}\right) - \frac{\tau}{N}.
\]

Otherwise, profit is zero: if the first inequality does not hold, inverse ad demand is zero, if the second inequality does not hold, broadcaster \(i\) has no viewers. Therefore, broadcaster \(i\) will choose an \(a_i > 0\) such that
\[
\frac{3\tau}{N\left(\frac{\sigma}{\beta m} + \delta\right)} > a_i > 0.
\]

(6)

We first consider the profit maximizing \(v_i\) for a given \(a_i\) satisfying (6). Note that \(\pi_i\) is strictly concave in \(v_i\). Solving the first order condition
\[
\frac{\partial\pi_i}{\partial v_i} = \frac{n}{\tau} \left(\sigma - \beta v_i - \frac{\sigma a_i}{m}\right) a_i - \beta n \left(\frac{1}{N} + \frac{v_i - \delta a_i - \left(\frac{\sigma}{\beta} - \frac{2\tau}{N}\right)}{\tau}\right) a_i = 0
\]
for \(v_i\) shows that the profit maximizing program quality is
\[
v_i^* (a_i) = \frac{1}{2} \left(\left(1 - \frac{a_i}{m}\right) \frac{\sigma}{\beta} + \delta a_i + \left(\frac{\sigma}{\beta} - \frac{2\tau}{N}\right) - \frac{\tau}{N}\right).
\]

Substituting \(v_i^* (a_i)\) into the profit of broadcaster \(i\) gives
\[
\pi_i (a_i, v_i^* (a_i)) = \frac{n\beta}{4\tau} \left(\frac{3\tau}{N} - a_i \left(\frac{\sigma}{\beta m} + \delta\right)\right)^2 a_i.
\]
The first order condition
\[
\frac{d}{da_i} \pi_i (a_i, v_i^* (a_i)) = 0
\]
has the solutions
\[
a_{i1} = \frac{3\tau}{N\left(\frac{\sigma}{\beta m} + \delta\right)}
\]
corresponding to the upper bound on \(a_i\) in (6), and
\[
a_{i2} = \frac{a_{i1}}{3} = \frac{m\beta \tau}{N\left(\sigma + m\beta \delta\right)}
\]
which is equation (11) in the paper. Moreover, it is straightforward to show that \(\pi_i (a_i, v_i^* (a_i))\) as a function of \(a_i\) is: zero at \(a_i = 0\), strictly concave when \(a_i < 2a_{i1}/3\), strictly convex when \(2a_{i1}/3 < a_i < a_{i1}\), and zero at \(a_i = a_{i1}\). It follows that \(a_{i2}\) maximizes profit. Noting that \(v_i^* (a_{i2})\) is the value of \(v_i\) given in the Proposition completes step 1.

**Step 2: Describe the range of \(a_i\) and \(v_i\) leading to undercutting.**

Suppose broadcaster \(j\) is a distance \(k/N\) away from broadcaster \(i\). A consumer with ideal point at the location of broadcaster \(j\) is indifferent between the products of broadcasters \(i\) and \(j\) if
\[
v_i - \delta a_i - \frac{k\tau}{N} = v_j - \delta a_j.
\]

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Consider a unilateral deviation of broadcaster $i$, while all broadcasters except $i$ stick to the equilibrium strategies, i.e., $v_j - \delta a_j = \frac{\sigma}{\beta} - \frac{2\tau}{N}$. Thus, the consumer is indifferent if
\[
v_i - \delta a_i = \frac{\sigma}{\beta} - \frac{2\tau}{N} + \frac{k\tau}{N}.
\]
For $k = 1, 2, \ldots$, the values of $(a_i, v_i)$ satisfying this equation are the points of discontinuity of the demand of broadcaster $i$. Note that the discontinuity at $k = 1, 2, \ldots$ corresponds to just undercutting $2k$ rivals. For simplicity and w.l.o.g., we break all ties in favor of broadcaster $i$, i.e., we assume that if a broadcaster deviates then any consumer that is indifferent between the deviating and another broadcaster watches the deviating broadcaster.

To summarize, broadcaster $i$ undercuts no rival if
\[
v_i - \delta a_i < \frac{\sigma}{\beta} - \frac{2\tau}{N} + \frac{\tau}{N}.
\]
Broadcaster $i$ undercuts exactly $2k$ rivals, $k = 1, 2, \ldots$, if
\[
\frac{\sigma}{\beta} - \frac{2\tau}{N} + \frac{k\tau}{N} \leq v_i - \delta a_i < \frac{\sigma}{\beta} - \frac{2\tau}{N} + \frac{(k + 1)\tau}{N}.
\]

**Step 3:** Broadcaster $i$ cannot make a positive profit by undercutting more than 2 broadcasters.

By the symmetry of the behavior of broadcasters $j \neq i$, if broadcaster $i$ undercuts more than 2 broadcasters, it undercuts 4 or more broadcasters. To undercut 4 or more broadcasters, the program quality of broadcaster $i$ needs to be $v_i \geq \sigma/\beta + \delta a_i$. However, then $\sigma - \beta v_i - \sigma a_i/m < 0$, thus inverse ad demand is zero.

**Step 4:** Undercutting two rivals: the case $N = 3$.

Suppose that broadcaster $i$ undercuts exactly 2 rivals. That is,
\[
\frac{\sigma}{\beta} - \frac{\tau}{3} \leq v_i - \delta a_i < \frac{\sigma}{\beta}.
\]
If $N = 3$, this means broadcaster $i$ has a market share of 1. The profit of broadcaster $i$ is then
\[
n \left( \sigma - \beta v_i - \frac{\sigma a_i}{m} \right) a_i. \tag{7}
\]
Note this is strictly decreasing in $v_i$, therefore, the optimal undercutting of two rivals satisfies
\[
\frac{\sigma}{\beta} - \frac{\tau}{3} = v_i - \delta a_i.
\]
Solving for $v_i$ gives
\[
v_i = \hat{v}_i (a_i) := \frac{\sigma}{\beta} - \frac{\tau}{3} + \delta a_i,
\]
substituting in (7) shows that the profit is
\[
\pi_i^{\text{dev}} (a_i, \hat{v}_i (a_i)) = n \left( \sigma - \beta \left( \frac{\sigma}{\beta} - \frac{\tau}{3} + \delta a_i \right) - \frac{\sigma a_i}{m} \right) a_i.
\]
Note $\pi_i^{\text{dev}}(a_i, \hat{v}_i(a_i))$ is strictly concave in $a_i$. The first order condition
\[
\frac{d}{da_i} \pi_i^{\text{dev}}(a_i, \hat{v}_i(a_i)) = 0
\]
has the unique solution
\[
a_i^{\text{dev}} = \frac{m\beta\tau}{6(\sigma + m\beta\delta)}.
\]
Moreover,
\[
\hat{v}_i(a_i^{\text{dev}}) = \frac{\sigma}{\beta} - \frac{\tau}{3} + \delta \frac{m\beta\tau}{6(\sigma + m\beta\delta)}.
\]
The profit from the deviation is
\[
\pi_i^{\text{dev}}(a_i^{\text{dev}}, \hat{v}_i(a_i^{\text{dev}})) = \frac{3}{4} \frac{nm\beta^2 \tau^2}{27(\sigma + m\beta\delta)},
\]
which is $3/4$ of the equilibrium profit given in the Proposition.

It follows that in case $N = 3$, broadcaster $i$ has no incentive to undercut two rivals.

**Step 5:** In case $N > 3$, from all strategies involving undercutting two rivals, the best strategy is at a point of discontinuity of demand.

Suppose broadcaster $i$ undercuts exactly 2 rivals:
\[
\frac{\sigma}{\beta} - \frac{\tau}{N} \leq v_i - \delta a_i < \frac{\sigma}{\beta}.
\]
If $N > 3$, broadcaster $i$ has a market share of
\[
\frac{3}{N} + \frac{v_i - \delta a_i - \left(\frac{\sigma}{\beta} - \frac{\tau}{N}\right)}{\tau}.
\]
Profit is
\[
\pi_i^{\text{Dev}}(a_i, v_i) = n \left(\frac{3}{N} + \frac{v_i - \delta a_i - \left(\frac{\sigma}{\beta} - \frac{\tau}{N}\right)}{\tau}\right) \left(\sigma - \beta v_i - \frac{\sigma a_i}{m}\right) a_i.
\]
We will show that this profit is maximal whenever $i$ just undercuts two rivals, i.e., when
\[
\frac{\sigma}{\beta} - \frac{\tau}{N} = v_i - \delta a_i.
\]\(8\)
To see this, suppose that broadcaster $i$ undercuts 2 rivals, and
\[
\frac{\sigma}{\beta} - \frac{\tau}{N} < v_i - \delta a_i < \frac{\sigma}{\beta}.
\]
For a fixed $a_i$,
\[
\frac{\partial}{\partial v_i} \pi_i^{\text{Dev}} = \left(\frac{1}{\tau} \left(\sigma - \beta v_i - \frac{\sigma a_i}{m}\right) - \beta \left(3 \frac{v_i - \delta a_i - \left(\frac{\sigma}{\beta} - \frac{\tau}{N}\right)}{\tau}\right)\right) na_i.
\]
Note that $\pi_i^{Dev}$ is strictly concave in $v_i$. Moreover, at $v_i = \frac{\sigma}{\beta} - \frac{\tau}{N} + \delta a_i$, we have
\[
\frac{\partial}{\partial v_i} \pi_i^{Dev} \bigg|_{v_i=(\frac{\sigma}{\beta} - \frac{\tau}{N} + \delta a_i)} = \left( \frac{1}{\tau} \left( \frac{\beta \tau}{N} - \beta \delta a_i - \frac{\sigma a_i}{m} \right) - \beta \frac{3}{N} \right) na_i
\]
\[
= - \left( \frac{1}{\tau} \left( \beta \delta a_i + \frac{\sigma a_i}{m} \right) + \beta \frac{2}{N} \right) na_i < 0.
\]
Therefore, in the relevant range $\pi_i^{Dev}$ is strictly decreasing in $v_i$ for fixed $a_i$. It follows that the best strategy involving undercutting two rivals must satisfy equation (8).

**Step 6:** In case $N > 3$, broadcaster $i$ has no incentive to just undercut 2 rivals.

Suppose broadcaster $i$ just undercuts 2 rivals, i.e., equation (8) holds. Then
\[
\pi_i^{Dev} = n \frac{3}{N} \left( \sigma - \beta v_i - \frac{\sigma a_i}{m} \right) a_i.
\]
Solve equation (8) for $v_i = \frac{\sigma}{\beta} - \frac{\tau}{N} + \delta a_i$ and substitute into $\pi_i^{Dev}$ to get
\[
\pi_i^{Dev} = n \frac{3}{N} \left( \sigma - \beta \left( \frac{\sigma}{\beta} - \frac{\tau}{N} + \delta a_i \right) - \frac{\sigma a_i}{m} \right) a_i.
\]
Note that this is a strictly concave function of $a_i$. The first order condition
\[
\frac{\partial}{\partial a_i} \pi_i^{Dev} = 0
\]
has the unique solution
\[
a_i = \frac{1}{2N} \frac{m \beta}{\sigma + m \beta \delta} \frac{\tau}{\delta}.
\]
Inserting this into $\pi_i^{Dev}$ gives
\[
\pi_i^{dev} = \frac{3}{4N^3} \frac{nm \beta^2 \tau^2}{(\sigma + \beta \delta)^2}
\]
which is $3/4$ of equilibrium profit. It follows that broadcaster $i$ has no incentive to just undercut two rivals.

**References**


